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Two-dimensional superstrings and the supersymmetric matrix model

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Abstract: We present evidence that the supersymmetric matrix model of Marinari and Parisi represents the world-line theory of N unstable D-particles in type II superstring theory in two dimensions. This identification suggests that the matrix model gives a holographic description of superstrings in a two-dimensional black hole geometry.

Keywords: Matrix Models, Tachyon Condensation, D-branes.

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1. Introduction

Matrix models provide an elegant and powerful formalism for describing low-dimensional string theories. Recently, it was proposed that the large N matrix variables can be viewed as the modes of N unstable D-particles in the corresponding string theory, in a decoupling limit [1, 2]. This proposal reinterprets the matrix-model/string-theory correspondence as a holographic open-string/closed-string duality, and suggests a search algorithm for more examples. It has been clarified [3]–[6] and very recently extended to type 0 strings [7, 8]. In this note, we apply this perspective to shed some new light on the physical identification of the supersymmetric matrix model of Marinari and Parisi [9].

The paper is organized as follows. We begin by recollecting the basic features of the Marinari-Parisi model and its proposed continuum limit. In section 3, we review some of the target space properties of 2-d superstring theory. In section 4 we collect a list of correspondences between the two theories. Most notably, we find that the open string spectrum on unstable D-particles of the 2-d string theory is that of (a minor improvement of) the MP model, expanded around the maximum of its potential. We also make a direct comparison between the vacuum structure and instantons of both models. We end with some concluding remarks and open problems. Some technical discussions are sequestered to appendices A and B.

2. The Marinari-Parisi model

The Marinari-Parisi model is the quantum mechanics of an $N \times N$ hermitean matrix in a one-dimensional superspace

$$\Phi(\tau,\theta,\overline{\theta}) = M(\tau) + \overline{\theta}\Psi(\tau) + \overline{\Psi}(\tau)\theta + \theta\overline{\theta}F(\tau)\,. \tag{2.1}$$

The action is

$$S = -N \int d\tau\, d\overline{\theta}\, d\theta\, \mathrm{Tr}\left\{\frac{1}{2}\overline{D}\Phi D\Phi + W_0(\Phi)\right\}, \tag{2.2}$$

where $D, \overline{D}$ are superspace derivatives. We can choose a cubic superpotential

$$W_0(\Phi) = \frac{1}{2}\Phi^2 - \frac{1}{3}\lambda^2\Phi^3\,. \tag{2.3}$$

The Feynman graph expansion for the model generates a discretization of random surfaces in superspace. Related work on supersymmetric matrix models includes [10]–[20]

The model we will discuss is actually a slight modification of the original MP model. We will take the derivatives appearing in (2.2) to be covariant with respect to gauge transformations which are local in superspace; their form is described in appendix A. In the case of the $c = 1$ matrix model, its identification with the worldline theory of D-particles made clear that the $\mathrm{U}(N)$ conjugation symmetry of the matrix model should be gauged. As we will see, the same correspondence in our case suggests that we should introduce a superfield gauge symmetry in the model (2.2), which naturally effects the truncation to singlet states [12].

The model with superpotential (2.3) has two classical supersymmetric extrema $W_0'(\Phi) = 0$. These are minima of the bosonic potential $V(M) = M^2(1-\lambda^2 M)^2$. In addition, V has an unstable critical point at $M_c = 1/2\lambda^2$. The quadratic form of the action, when expanded near this non-supersymmetric critical point is (defining $Y = M - M_c$)

$$S = -N \int d\tau\, \mathrm{Tr}\left\{\frac{1}{2}(D_\tau Y)^2 + \overline{\Psi}D_\tau\Psi + \frac{1}{2}\lambda^2 Y^2 - \frac{1}{16\lambda^2}\right\}. \tag{2.4}$$

In the following we will argue that this action can be viewed as that of N unstable D-particles, localized in the strong coupling/curvature region of the 2d string theory background.

In the MP model, the fermi level is not an independent parameter, in that it is determined by the form of the potential [12]. Critical behavior arises instead in this model through a singularity in the norm of the ground-state wavefunction [12]. We will discuss the ground states of the matrix model further in section 4, but for now it suffices to study an exemplary one, $|f_0\rangle$, whose norm is given by

$$e^{\mathcal{F}} \equiv |f_0|^2 = \int \prod_i dz_i \prod_{i<j}(z_i - z_j)^2\; e^{-2W_0(z)}\,; \tag{2.5}$$

this is a $c < 1$ matrix integral. For odd W_0, there is an irrelevant divergence at large $|z|$ which we simply cut off. A critical limit arises by tuning the potential W_0 to the $m = 2$ pure-gravity critical point of [21], near which the tree-level free energy is $\mathcal{F} \propto \kappa^{-2}$, with $\kappa^{-1} = (\lambda - \lambda_c)^{5/4}N$ providing the string coupling.

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This limit naively gives a supersymmetric sigma model on 1d superspace $(\tau, \theta, \overline{\theta})$ coupled to 2d Liouville supergravity. In [22], however, the following argument was presented against such a description of the continuum limit: the matter part of the action is necessarily interacting, and has a one-loop beta function predicting that the coupling grows in the IR and that the matter fields become disordered. This would seem to indicate that the superspace coordinates $(\tau, \theta, \overline{\theta})$ become massive, and that spacetime does not survive in the critical theory. We consider this conclusion premature. The reasoning assumes that the matter theory and the worldsheet gravity are coupled only via the gauge constraints. This is not the case for the supersymmetric string in two dimensions [23].

3. Two-dimensional superstrings

To formulate 2d superstring theory, one starts from $\mathcal{N} = 2$ Liouville theory [24] and then performs a consistent GSO projection to obtain a string theory with target space supersymmetry [23, 25]. Unlike bosonic and $\mathcal{N} = 1$ supersymmetric Liouville theory, the time direction τ is involved in the $\mathcal{N} = 2$ supersymmetry algebra, and it is involved in the $\mathcal{N} = 2$ Liouville interaction as well:

$$\mathcal{L}_{int}^{SL} = \psi\widetilde{\psi}\, e^{-\frac{1}{2}(\rho+\widetilde{\rho}+i(\tau-\widetilde{\tau}))} + \text{c.c.} \tag{3.1}$$

with $\psi = \psi_\rho + i\psi_\tau$.

Interestingly, $\mathcal{N} = 2$ Liouville theory has been shown [26] to be dual to superstrings propagating inside the 2d black hole defined by the supercoset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ [27]. The semiclassical background is

$$\begin{aligned} ds^2 &= d\rho^2 + \tanh^2\rho\, d\tau^2\,, \qquad \tau \equiv \tau + 2\pi\,; \\ \Phi &= \Phi_0 - \log\cosh\rho\,, \end{aligned} \tag{3.2}$$

with $g_s = e^{\Phi}$.[1] This explicitly shows that in the infrared region of small e^ρ, the τ direction degenerates. The dependence of the string background on ρ can be attributed to a gravitational dressing of the operators. This does however not preclude the existence of a two-dimensional continuum description (c.f. the above discussion of the scaling of the MP matrix model).

We now summarize a few properties of the target space theory. A good starting point is the worldsheet description of the euclidean theory, the fermionic cigar at the free-fermion radius [25]. The string worldsheet theory on the cigar has three conserved currents: the left-moving and right-moving chiral currents J and $\widetilde{J}$, with

$$J = -\overline{\psi}\psi + i2\partial\tau \equiv i\partial(H + 2\tau)\,, \tag{3.3}$$

and a non-chiral current whose integral charge P_τ is the quantized euclidean energy, i.e. the discrete momentum around the cigar. The chiral projection that defines the type II theories is the condition that physical operators should have a local OPE with the spectral

[1] Type IIB string theory based on this CFT is also equivalent, via a more trivial T-duality, to type IIA string theory on the circle of the inverse radius, with (3.1) replaced by the corresponding momentum condensate.

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flow operators, which in type IIB string theory takes the form

$$S = e^{-\frac{1}{2}\varphi + \frac{i}{2}(H+2\tau)} \qquad \widetilde{S} = e^{-\frac{1}{2}\widetilde{\varphi} + \frac{i}{2}(\widetilde{H}+2\widetilde{\tau})} \tag{3.4}$$

(here φ denotes the bosonized superghost current). Since the U(1) R current of the $\mathcal{N}=2$ algebra involves the compact boson τ in addition to the worldsheet fermions, the symmetry generator P_τ is an R symmetry (here $\mathcal{S} = \int dz\ S$ is the supercharge)

$$[P_\tau, \mathcal{S}] = \frac{1}{2}\mathcal{S} \qquad\qquad [P_\tau, \widetilde{\mathcal{S}}] = \frac{1}{2}\widetilde{\mathcal{S}}\,. \tag{3.5}$$

States in a given supersymmetry multiplet therefore do not all have the same energy.

The perturbative closed string spectrum in the euclidean IIB string theory consists [25] of an NSNS (non-tachyonic) tachyon with odd winding modes, a left-moving periodic RR scalar (the self-dual axion), and a right moving complex fermion Υ with half-integer momenta. This is the expected behaviour for spinors which are single valued on the cigar. Therefore, in the compact theory, a rotation $\tau \to \tau + 2\pi$ acts on the spacetime fields as $e^{2\pi i P_\tau} = (-1)^{F_s}$, where F_s is the target-space fermion number. There is another $\mathbb{Z}_2$ symmetry $(-1)^{F_L}$ which acts by

$$\chi \mapsto -\chi\,, \qquad \Upsilon \mapsto \overline{\Upsilon}\,. \tag{3.6}$$

The $\mathcal{N}=2$ Liouville which has primarily been considered in the literature has a euclidean time direction. On the other hand, matrix quantum mechanics is most easily described in a real-time hamiltonian language. It will therefore be convenient for us to hypothesize a consistent analytic continuation of this theory. However, this analytic continuation needs to be understood better. Translating to a minkowskian spectrum, we find a left-moving scalar χ and a complex right-moving fermion Υ. In addition to these propagating degrees of freedom, there are also discrete physical states at special energies.

D-Instantons and flux sectors. The physics of the RR axion is closely linked to that of D-instantons. Two-dimensional IIB string theory, however, has some special features. First, the axion is a self-dual middle-rank form; it couples both electrically and magnetically to the D-instanton. One important implication of this is that the axion itself does not have a well-defined constant zeromode. Secondly, unlike the 10d case, where the BPS D-instanton breaks sixteen supercharges and thus carries an even number of fermion zeromodes, it seems that the 2d D-instanton only breaks *one* supersymmetry and therefore carries only one fermion zeromode. It thus interpolates between sectors with opposite fermion parity. A preliminary study of the D-instanton boundary state in appendix B bears this out.

At this point it is natural to introduce the right-moving scalar field U via bosonization $\Upsilon = e^{iU}$. In this bosonized language, the entire field-theoretic spectrum of 2d type IIB can thus be reassembled into a single non-chiral scalar field $\phi = \phi_L + \phi_R$ with

$$\phi_L = \chi\,, \qquad \phi_R = U\,. \tag{3.7}$$

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Since $\phi_R = U$ is periodic with the free-fermion radius, it is natural to suspect that the axion $\phi_L = \chi$ is periodic with the free-fermion radius as well.[2] Note that $(-1)^{F_L}$ as defined above acts by $(-1)^{F_L} : \phi \mapsto -\phi$.

The fact that the D-instanton has only one fermion zero mode means the operator that creates it carries fermion number 1. This indicates that $\oint_\gamma \overline{\partial} U = 1$, where γ is a contour containing the instanton. Further, from the coupling of the D-instanton to the RR axion, we expect that in the presence of a D-instanton $\oint_\gamma \partial\chi = 1$. A natural candidate for the effective operator with the right properties to create a D-instanton at the space-time location x is then

$$e^{i\phi(x)} = e^{i(\chi(x)+U(x))} . \tag{3.8}$$

Instantons are tunnelling events that interpolate between perturbative sectors. These sectors are characterized by an integer flux (here Σ denotes a space-filling contour):

$$\int_\Sigma \partial_0 \phi = k \qquad k \in \mathbb{Z} , \tag{3.9}$$

which is the quantized momentum dual to the constant zero mode ϕ_0 of ϕ (ϕ_0 is periodic with period 2π). The integer k can be thought of as a slight generalization of the "s-charge" of [29].[3]

4. Dual correspondence

We will now try match the physics of the Marinari-Parisi matrix model with that of the two-dimensional type IIB string theory. Following the logic of [1, 3] we start by examining the open string spectrum of the unstable D-particles.

D-particles. In type II string theory, the boundary state for an unstable Dp-brane has the form [31]–[34]

$$|\hat{D}p\rangle = |B, NSNS; +\rangle - |B, NSNS; -\rangle . \tag{4.1}$$

Here $|B, NSNS; \eta\rangle$ denotes a boundary state in the NSNS sector, satisfying $(G_r - i\eta\tilde{G}_r)$ $|B;\eta\rangle = 0$; $G = G^+ + G^-$ is the gauged worldsheet supercurrent. The boundary state describing an unstable brane with unperturbed tachyon contains no term built on Ramond primaries. Experience with less supersymmetric Liouville models suggests that branes localized in the Liouville direction correspond to boundary states associated with the Liouville vacuum state, which we will call $|B_0;\eta\rangle$. The defining property of these states is that the corresponding bosonic open string spectrum (of NS-sector open strings for which both end-points satisfy this specific boundary condition) have support only at Liouville momentum $P = -i$, corresponding to the identity Liouville state.

Details regarding these boundary states appear in appendix B. Using the general formula (4.1), the basic unstable D0-brane of type IIB is represented by the boundary state $|\hat{D}0\rangle = |B_0; +\rangle - |B_0; -\rangle$. Study of the annulus amplitude for this D-brane, detailed

[2] This claim can be verified (or refuted) by computing the axion charge carried by a D-instanton [28], or the axion flux produced by a decaying D-brane, along the lines of [3, 8].

[3] Since $\partial_+\phi = \partial_+\chi$, sectors with nonzero p_L are backgrounds in which flux quanta of the RR axion are turned on. Backgrounds of two-dimensional type IIA strings with RR flux were described in [30].

in appendix B, reveals that the open string spectrum on this brane is precisely that of the Marinari-Parisi model, expanded as in (2.4), including the gauge supermultiplet. This correspondence is the first strong indication that the MP model describes the type IIB non-critical string theory.

Symmetry considerations. There are two prominent continuous symmetries of the MP model: there is the conserved energy H, and there is the overall fermion number $\hat{F} \equiv \sum_i \psi_i^\dagger \psi_i$. In the Hilbert space of the MP model, the quantum number F takes N different values, for which we take the CP-invariant choice $-N/2, \ldots, N/2$. We would like to identify $(-1)^{F_s}$ of the target space theory with $(-1)^F$ of the MP model. Further, as in the bosonic and type 0 cases, we identify the hamiltonians of the systems

$$H = P_\tau \,.$$

The matrix model can also have a $\mathbb{Z}_2$ R-symmetry. Its interpretation can be understood as follows. Due to the coupling between the D-brane worldvolume fields and the closed strings, the worldvolume fields transform under $(-1)^{F_L}$. $(-1)^{F_L}$ acts by [32, 35]

$$Y \leftrightarrow -Y\,, \qquad \psi \leftrightarrow \psi^\dagger\,. \tag{4.2}$$

We will see below that this is consistent with $\Upsilon \leftrightarrow \Upsilon^\dagger$. Therefore $(-1)^{F_L}$ acts as an R-symmetry in the matrix quantum mechanics: it acts on the superspace coordinates as $(-1)^{F_L} : \theta \leftrightarrow \overline{\theta}$. In order for this to be a symmetry of the worldline action, W_0 *must be an odd function of* Y. This implies that supersymmetry is broken, since then none of the standard [36] candidate supersymmetric ground states $e^{\pm W}|0\rangle$ is normalizible.

Spectrum and $c = 1$ scaling. In [12], it was shown that the supercharges act within the space of super matrix eigenvalues as

$$Q = \sum_k \psi_k^\dagger \left(\frac{1}{N}\frac{\partial}{\partial z_k} + \frac{\partial W_{\text{eff}}(z)}{\partial z_k} \right), \qquad Q^\dagger = \sum_k \psi_k \left(\frac{1}{N}\frac{\partial}{\partial z_k} - \frac{\partial W_{\text{eff}}(z)}{\partial z_k} \right), \tag{4.3}$$

where

$$W_{\text{eff}}(z) = \sum_k W_0(z_k) - \frac{1}{N}\sum_{k<l} \log(z_k - z_l)\,. \tag{4.4}$$

These supercharges exactly coincide [37] with those of the supersymmetric Calogero-Moser model [38, 39]. The corresponding hamiltonian reads

$$\mathcal{H} = \sum_{i=1}^{N} \left(\frac{1}{2}p_i^2 + V(z_i) + \frac{2}{N} W_0''(z_i)\psi_i^\dagger \psi_i \right) + \frac{1}{N^2}\sum_{i<j} \frac{1-\kappa_{ij}}{(z_i - z_j)^2}\,, \tag{4.5}$$

where with $p_i = \frac{-i}{N}\frac{\partial}{\partial z_i}$ and $\kappa_{ij} = 1 - (\psi_i - \psi_j)(\psi_i^\dagger - \psi_j^\dagger)$ is the fermionic exchange operator [39]; it assigns fermi-statistics to the spin-down eigenvalues, and bose-statistics to the spin-up ones (here we are using the terminology of appendix A). The potential reads [12]

$$V(z) = \frac{1}{2}\left(W_0'(z) \right)^2 - W_0''(z)\,, \tag{4.6}$$

where we used that for a cubic superpotential W_0 one has $\sum_{i<j} \frac{W_0'(z_i)-W_0'(z_j)}{z_i-z_j} = (N-1) \times \sum_i W_0''(z_i)$. We see that the hamiltonian describes a system of interacting eigenvalues. The interaction is such that eigenvalues always repel each other: it represents a $2/r^2$ repulsion between the boson states with $\kappa_{ij} = -1$, and although it vanishes between two fermionic states with $\kappa_{ij} = 1$, such particles still avoid each other since their wavefunctions are antisymmetric.

In the case all the eigenvalues have spin down, so that all $\kappa_{ij} = 1$, the hamiltonian reduces to a decoupled set of one-particle hamiltonians. It is easy to write the ground state wave function in this case. Let us introduce the notation $\Delta(i_1,..i_k) = \prod_{i<j\in(i_1,..i_k)}(z_{ij})$, the vandermonde of the k variables $\{z_{i_k}\}$ (here $z_{ij} = z_i - z_j$). In this notation:

$$|f_0\rangle \equiv f_0|\downarrow\downarrow\cdots\downarrow\rangle = e^{\mathrm{Tr}\,W_0}\Delta(1,2..n)|\downarrow\downarrow\cdots\downarrow\rangle\,. \tag{4.7}$$

This vacuum state represents the filled Fermi sea of the first N energy levels. In the harmonic potential, this is a supersymmetric ground state. In the cubic potential, there is a corresponding ground state in each well, only one of which is perturbatively supersymmetric.

In the sectors with non-zero fermion number, there are no normalizable supersymmetric ground states, even for the harmonic well. In this case, one can show that the groundstate eigenfunction in the fermion number k sector is:

$$|f_k\rangle = \sum_{i_1<i_2<\cdots<i_k} \Delta(i_1,\ldots,i_k)\prod_{m=1}^{k}\psi^\dagger_{i_m}|f_0\rangle\,. \tag{4.8}$$

This wavefunction obeys the right statistics imposed by gauge invariance. From the expression (4.8) we can read off that the energy levels of the spin up eigenvalues are double spaced relative to that between spin down states [40]. Now let us discuss the system with a cubic superpotential. The state f_0 is a perturbatively supersymmetric state, with a filled fermi sea in the left well. It turns out that the fermi level exactly coincides with the bottom of the second well, as indicated on the left of figure 1. Although the super-Calogero-Moser model with a cubic potential has no known exact treatment, it seems reasonable to assume that the super-eigenvalues in each of the two potential wells behave qualitatively similar as for the single harmonic potential. This suggests that in the sector with k up-spins, the ground state is well-approximated by taking the single particle energy spectrum, and fill the first $N-k$ levels with the fermionic eigenvalues, and all levels $N-k+2m$ with $m = 1,\ldots,k$ up to the $N+k$-th energy level. When k gets large enough, this $N+k$-th energy level starts approaching the unstable maximum of the potential, as indicated on the right of figure 1. Here we expect to find $c = 1$ critical behavior. Our proposal is that the double scaled eigenvalue dynamics near this unstable maximum encodes the scattering non-perturbative physics of non-critical IIB strings.

Matrix model instantons. Single-eigenvalue tunneling events in the matrix model interpolate between the sectors with different number of up-spins: they relate the adjacent ground states $|f_k\rangle$ and $|f_{k+1}\rangle$. Note that these sectors have opposite parity of fermion

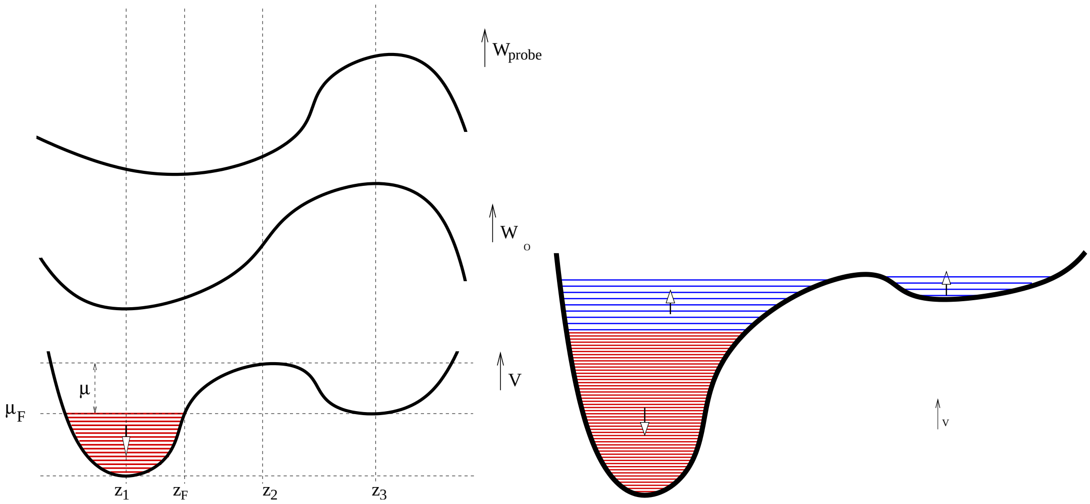

Figure 1: The approximate probe superpotential, superpotential and bosonic potential of the f_0 state. The fermi level of the perturbatively supersymmetric ground state f_0 coincides with the minimum of the right well (*left*). The ground state in the sector with k spin up eigenvalues fills up to the $N+k$-th energy level, which in the double scaling limit approaches the unstable maximum (*right*).

number. The probe super-eigenvalue Z moves in the mean-field superpotential

$$W_{\text{probe}}(Z) = W_0(Z) - \frac{1}{2}\sum_i \ln(Z - X_i)\,. \tag{4.9}$$

There exists [12] a BPS tunneling trajectory:

$$0 = \delta\psi = \dot{z}_{cl} - W'_{\text{probe}}(z_{cl})\,. \tag{4.10}$$

This trajectory breaks just one supersymmetry, and thus supports a single fermion zero mode.

We propose to identify the vacuum of the matrix model with fermion number k with the string theory vacuum with k units of flux, as defined in section 3. This identification is directly supported by the interpretation of the MP matrix model as the worldline theory of the unstable branes and of the D-instantons with the tunnelling trajectories of the worldline tachyon field Y. Recall that an unstable type II Dp-brane couples to the RR p-form potential $C_{(p)}$ via the gradient of its tachyon according to $\int C_{(p)} \wedge d\mathcal{W}(Y)$ [33] and [41]–[43] with $V(Y) = \frac{\partial}{\partial Y}\mathcal{W}(Y)$ [44]. For the unstable D-particle, this takes the form

$$S_{\hat{D0}} \supset \int \chi\, \frac{dY}{dt}\, V(Y)\; dt\,. \tag{4.11}$$

Combined with our proposal, this coupling implies that the tunnelling trajectory sources the RR axion. It can therefore be identified with a D-instanton, further vindicating the prescient analysis of [45].

5. Concluding remarks

We have collected evidence supporting the conjecture that the supersymmetric matrix model of Marinari and Parisi can be identified with the matrix mechanics of N unstable D-particles in two-dimensional IIB string theory. This suggests that in a suitable double scaling limit, the MP model, when viewed from this perspective, provides a non-perturbative definition of the string theory. The two systems on both sides of the conjectured duality, however, clearly need further study. We end with some concluding comments.

Space-time fields. An important open problem is the proper identification of the space-time fields in the MP model. It is reasonable to expect that, as for the $c = 1$ [46] and $\hat{c} = 1$ cases, the spacetime fields arise from the matrix model via the collective fields for the eigenvalue density. Possibly the supersymmetric collective field theory of [47] and [13]–[15] is the correct framework, though it seems that some Z_2 projection may be needed, since the target space boson χ and fermions Υ are chiral with opposite chirality. If Υ is linear in the fermionic component of the eigenvalue density (e.g. a Laplace transform of it), then the matrix model action of $(-1)^{F_L} : \psi \leftrightarrow \overline{\psi}$, is consistent with the action on closed-string fields $\Upsilon \leftrightarrow \overline{\Upsilon}$.

Space-time supersymmetry. Space-time supersymmetry should provide a helpful guideline in finding a precise dictionary. Expanded around the quadratic maximum of its bosonic potential, the model (2.4) has many symmetries; indeed the small bosonic and fermionic fluctuations are decoupled. It is not difficult to find fermionic operators which behave as in (3.5). The more mysterious question is how to describe the matrix model supersymmetry in the target space of the string theory.

D-brane decay. It should be possible to generalize the analysis of [1, 3, 4] to study the decay of a single unstable D-brane. This will presumably involve a superfield version of the fermion operator that creates and destroys the super-eigenvalues, and a superfield bosonization formula along the lines of [48]. This analysis would allow an independent determination of the compactification radius of the axion, along the lines of [3, 8], confirming that χ is periodic at the free fermion radius.

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A. The gauged Marinari-Parisi model

In this appendix, we describe two ways of introducing an auxiliary gauge field for the supersymmetric matrix model. We show that the second method is equivalent to the eigenvalue reduction of the MP model given in [12].

Gauged model, version I. The conventional method of gauging a supersymmetric action is to introduce a real matrix superfield $\mathcal{V}$, and replace the superderivatives in (2.2) with gauge-covariant superderivatives of the form

$$D_{\mathcal{V}}\Phi = e^{\mathrm{ad}\mathcal{V}} D(e^{-\mathrm{ad}\mathcal{V}}\Phi)\,, \qquad D = \partial_\theta + \overline{\theta}\partial_\tau \tag{A.1}$$

$$\overline{D}_{\mathcal{V}}\Phi = e^{\mathrm{ad}\mathcal{V}} \overline{D}_\theta(e^{-\mathrm{ad}\mathcal{V}}\Phi)\,, \qquad \overline{D} = \partial_{\overline{\theta}} + \theta\partial_\tau\,. \tag{A.2}$$

These derivatives are designed to be covariant under local gauge transformations

$$\Phi \mapsto e^{\mathrm{ad}\Lambda}\,\Phi\,, \qquad \mathcal{V} \mapsto \mathcal{V} + \Lambda\,, \tag{A.3}$$

where Λ is an arbitrary real superfield. We can thus choose the gauge $\mathcal{V} = 0$. The presence of the gauge field still manifests itself, however, by means of the requirement that physical states must be annihilated by the generator of infinitesimal *bosonic* gauge rotations $\Phi \mapsto U^\dagger \Phi U$. This invariance can be used to diagonalize, say, the bosonic component ϕ of the matrix superfield. The off-diagonal fermionic matrix elements, however, remain as physical degrees of freedom.

Gauged model, version II. A second possible procedure is to introduce a complex superfield $\mathcal{A}$ and define superderivatives

$$D_{\mathcal{A}}\Phi = D\Phi - [\mathcal{A}, \Phi]\,, \qquad \overline{D}_{\mathcal{A}}\Phi = \overline{D}\Phi - [\overline{\mathcal{A}}, \Phi] \tag{A.4}$$

covariant under local gauge transformations

$$\Phi \mapsto e^{\mathrm{ad}\Lambda}\,\Phi\,, \qquad \mathcal{A} \mapsto \mathcal{A} + D\Lambda\,, \qquad \overline{\mathcal{A}} \mapsto \overline{\mathcal{A}} + \overline{D}\Lambda \tag{A.5}$$

with Λ an arbitrary real matrix superfield. In this case, the gauge invariance is not sufficient to choose a gauge in which $\mathcal{A}$ and $\overline{\mathcal{A}}$ are set equal to zero. However, since $\mathcal{A}$ and $\overline{\mathcal{A}}$ appear as non-dynamical fields, we can eliminate them via their equations of motion

$$\mathcal{G} \equiv [\Phi, \Pi] = 0\,, \qquad \Pi \equiv D_{\mathcal{A}}\Phi\,. \tag{A.6}$$

We will impose the physical state conditions in the weak form

$$\mathcal{G}\,|\Psi_{\mathrm{phys}}\rangle = 0\,. \tag{A.7}$$

The space of solutions to this constraint is characterized as follows. Let U be the bosonic unitary matrix that diagonalizes the bosonic component ϕ of the matrix superfield. We can then define

$$\left(U\Phi U^\dagger\right)_{kk} = z_k + \overline{\theta}\,\psi_k + \psi_k^\dagger\,\theta + \overline{\theta}\theta f_k\,. \tag{A.8}$$

Here the z_k are the eigenvalues of ϕ. A straightforward calculations shows that the gauge invariance conditions is solved by physical states that depend on z_k and ψ_k only. Since the physical state constraint (A.6) is a supermultiplet of constraints, this guarantees that this subspace forms a consistent supersymmetric truncation of the full matrix model. One can also verify directly that it is invariant under supersymmetry transformations. After absorbing a factor of $\Delta = \prod_{i<j}(z_i - z_j)$ into our wave functions

$$\Psi(z,\psi) = \Delta(z)\tilde{\Psi}(z,\psi) \tag{A.9}$$

the supersymmetry generators take the form (4.3).

It is convenient to think of the system of eigenvalues as N particles moving in one dimension, each with an internal spin $\frac{1}{2}$ degree of freedom, a spin "up" or "down." Accordingly, we can define the Hilbert space on which the fermionic eigenvalues act by

$$\psi_i| \downarrow\downarrow \cdots \downarrow\rangle = 0\,, \qquad \forall i\,; \qquad \psi_i^\dagger| \downarrow\downarrow \cdots \downarrow\rangle \equiv |\underbrace{\downarrow \cdots \downarrow}_{i-1}\uparrow\downarrow \cdots \downarrow\rangle\,; \qquad \text{etc.} \tag{A.10}$$

A general state in the physical Hilbert space is then

$$|f_\eta\rangle = \sum_\eta f_\eta(\mathbf{z})|\vec{\eta}\rangle\,, \tag{A.11}$$

where η is a vector of N up or down arrows, and we have arranged the eigenvalues in a vector $\mathbf{z}$. Since it is possible via $\mathrm{U}(N)$-rotations to interchange any eigenvalue superfield (z_i, ψ_i) with any other eigenvalue superfield (z_j, ψ_j), the matrix wavefunctions should be symmetric under this exchange operation.

However, since our wave-functions depend on anti-commuting variables, the model will inevitably contain bosonic as well as fermionic sectors. Let us now define a a fermionic interchange operation κ_{ij} with the property that it interchanges the i and j spin state, and also multiplies the overall wavefunction by a minus sign in case both spins point in the up-direction. This minus sign reflects the Fermi statistics of ψ_i and ψ_j. Define the total exchange operation as the product of the bosonic and fermionic one $\mathcal{K}_{ij} = K_{ij}\,\kappa_{ij}$ where K_{ij} interchanges z_i and z_j. We now specify the overall statistics of the physical wavefunctions by means of the requirement that

$$\mathcal{K}_{ij}\,|\tilde{\Psi}_{\rm phys}\rangle = -|\tilde{\Psi}_{\rm phys}\rangle \qquad\qquad \text{for all} \quad i,j\,. \tag{A.12}$$

The minus sign on the right-hand side ensures that the original wavefunction, before splitting off the Vandermonde determinant (see eqn. (A.9)), is completely symmetric. The condition (A.12) implies that particles with spin "up" are fermions, while particles with spin "down" are bosons. We can call this the spin-statistics theorem for our model.

B. Boundary states for $\mathcal{N} = 2$ Liouville

In this appendix, we will attempt to write down the boundary state for the unstable D-particle of type IIB in the $\mathcal{N} = 2$ Liouville background. In doing this, we will take advantage of the worldsheet $\mathcal{N} = 2$ supersymmetry by expanding the boundary state in Ishibashi states which respect the $\mathcal{N} = 2$. In order to write the Ishibashi states, we will need to recall some facts about the primaries on which they are built, and their characters.

Characters of the $\mathcal{N}=2$ superconformal algebra. The chiral $\mathcal{N}=2$ characters are defined by

$$\chi_V(q,y) = \mathrm{Tr}_V q^{L_0 - c/24} y^{J_0} \,. \tag{B.1}$$

The trace is over an $\mathcal{N}=2$ module V. These representations are built on primary states labelled by the eigenvalues h,ω of the central zeromodes L_0, J_0. It will be convenient to label our primary states by P and ω, related to the conformal dimension by $h = (Q^2/4 + P^2 + \omega^2)/2 = (1 + P^2 + \omega^2)/2$. The Liouville momentum P[4] is determined by this equation up to choice of branch, both of which have the same character.

These characters were written down in [49]. For the module associated with a generic NS primary, labelled by $[P,\omega]$, the character is

$$\chi^{NS}_{[P,\omega]}(q,y) = q^{P^2/2+\omega^2/2}\; y^{\omega} \frac{\vartheta_{00}(q,y)}{\eta^3(q)} \,. \tag{B.2}$$

In the R-sector, a primary is also annihilated by G_0^+ or G_0^-, and this results in an extra label $\sigma = \pm$ on the character. For the primary with labels P,ω,σ, the character is (let $y \equiv e^{2\pi i\nu}$)

$$\chi^{R}_{[P,\omega,\sigma]}(q,y) = 2\cos\pi\nu\; q^{P^2/2+\omega^2/2}\quad y^{\omega+\frac{\sigma}{2}} \frac{\vartheta_{1,0}(q,y)}{\eta^3(q)} \,. \tag{B.3}$$

We will also need the character for the identity representation

$$\chi^{NS}_{\mathbf{1}}(q,y) = q^{-1} \frac{1-q}{(1+yq^{1/2})(1+y^{-1}q^{1/2})} \frac{\vartheta_{0,0}(q,y)}{\eta^3(q)} \,. \tag{B.4}$$

Modular properties of the characters. The modular transformation properties of the chiral characters of the $\mathcal{N}=2$ algebra will be crucial for our study of D-branes in $\mathcal{N}=2$ superLiouville. We use the notation $\tilde{q} = e^{2\pi i\tau}$, $\tilde{y} = e^{2\pi i\nu}$ for closed string modular variables, and $q = e^{-2\pi i/\tau}$, $y = e^{\pi i\nu/\tau}$ for their open string transforms. The characters participate in the following formulas [50, 51].

$$\int_{-\infty}^{\infty} d\tilde{P} d\tilde{\omega} S(\tilde{p},\tilde{\omega})\; \chi^{NS}_{[\tilde{p},\tilde{\omega}]}(\tilde{q},\tilde{y}) = \chi^{NS}_{\mathbf{1}}(q,y) \tag{B.5}$$

$$\int_{-\infty}^{\infty} d\tilde{P} d\tilde{\omega} S(\tilde{p},\tilde{\omega}) \chi^{NS}_{[\tilde{p},\tilde{\omega}]}(\tilde{q},-\tilde{y}) \equiv \chi^{R}_{\mathbf{1}}(q,y) \tag{B.6}$$

$$S(\tilde{p},\tilde{\omega}) = \frac{\sinh^2 \pi\tilde{p}}{2\cosh(\pi p/2 + i\pi\omega/2)\cosh(\pi p/2 - i\pi\omega/2)} \,. \tag{B.7}$$

Note that these formulas (B.5)–(B.7) are relevant for the case that R-charge is not quantized, on which we focus for simplicity in our study of boundary states. The refinement of these formulas to the case of compact euclidean time follows from (B.5)–(B.6) by Fourier decomposition. Further, these formulas arise by performing a formal sum; a careful treatment of convergence issues reveals additional contributions from discrete states [51].

[4] Here we are defining Liouville momentum as P appearing in the wavefunction $e^{-(Q/2+iP)\rho}$.

Ishibashi states for $\mathcal{N}=2$. Using this notation for representations of the $\mathcal{N}=2$ algebra, let us now study Ishibashi states based on these representations.[5] Such states [55] provide a basis for D-brane states which respect the $\mathcal{N}=2$ algebra. They carry two kinds of labels: those which specify the primary of the chiral algebra on which the state is built; and those which specify the automorphism of the chiral algebra which was used to glue the left and right chiral algebras. In our notation, we will separate these labels by a semicolon. Only automorphisms of the $\mathcal{N}=2$ which preserve the gauged $\mathcal{N}=1$ subalgebra are allowed.

There is a $\mathbb{Z}_2$ automorphism group of the $\mathcal{N}=1$ subalgebra ($G=G^+ + G^-$)

$$G \to \eta G \tag{B.8}$$

with $\eta=\pm 1$. There is an additional $\mathbb{Z}_2$ automorphism of the $\mathcal{N}=2$ algebra, which commutes with (B.8), and which is generated by

$$G^{\pm} \to G^{\pm\xi},\, J \to -\xi J$$

with $\xi = \pm$ [52] (the trivial map, $\xi=+1$ is B-type, the nontrivial map $\xi=-1$ is A-type).

Let j label the $\mathcal{N}=2$ primaries; it is a multi-index with three components:

$$j = \left[h, n, \left\{ \begin{array}{c} NS \\ R+ \\ R- \end{array} \right\} \right].$$

(Recall that an R primary is further specified by whether it is annihilated by G_0^+ or G_0^-.) An A-type Ishibashi state satisfies

$$(L_n - \tilde{L}_n)|j;A,\eta\rangle\rangle\,, \qquad (G_r^{\pm} - i\eta \tilde{G}_r^{\mp})|j;A,\eta\rangle\rangle = 0\,, \qquad (J_n - \tilde{J}_n)|j;A,\eta\rangle\rangle\,, \tag{B.9}$$

where r is half-integer moded if $|j\rangle$ is an NS primary, and integer moded if $|j\rangle$ is from an R sector.

In order to make type II D-branes from these states, we will need to know the action of the fermion number operators on them. In the NS sector [34],

$$(-1)^F|j,NS;\xi,\eta\rangle\rangle = -|j,NS;\xi,-\eta\rangle\rangle = (-1)^{\tilde{F}}|j,NS;\xi,\eta\rangle\rangle\,. \tag{B.10}$$

Note that the existence of the state with one value of η plus the conserved chiral fermion number implies the existence of the other. In the R sector, the action is more subtle because of the fermion zero modes. Since unstable branes may be built without using RR Ishibashi states, we will not discuss them further.

Next we need to know the matrix of inner products between these states. The inner product is defined by closed-string propagation between the two ends of a cylinder:

$$\langle\langle j_1;\xi_1\eta_1|D(\tilde{q},\tilde{y})|j_2;\xi_2\eta_2\rangle\rangle = \delta(j_1,j_2)\delta_{\xi_1,\xi_2}\chi_{j_1}(\tilde{q},\eta_1\eta_2\tilde{y})\,, \tag{B.11}$$

where $D(\tilde{q},\tilde{y})$ is the closed-string propagator, twisted by the R-current. The delta-function on primaries is obtained from the overlap of closed-string states:

$$\delta(j_1,j_2) \equiv \left(\langle j_1| \otimes \langle \tilde{j_1}|\right) |j_2\rangle \otimes |\tilde{j}_2\rangle\,. \tag{B.12}$$

[5] Useful references include [34] and [52]–[54].

The D0 boundary state. We would now like to describe the boundary state for the unstable D-particle. Since it is extended in the R-symmetry direction, it should be a B-type brane. To construct consistent boundary states for $\mathcal{N}=2$ Liouville, we will follow the strategy which was successful for bosonic Liouville [56]–[58], and for $\mathcal{N}=1$ Liouville [59, 60]. Basically, the consistent boundary states are Cardy states [58]; their wavefunctions can be written in terms of the modular matrix $U_j(i) = S^i_j/\sqrt{S^i_0}$ [61]. To be more precise, suppose, as in [56], that we can expand the desired boundary state in Ishibashi states for the non-degenerate (NS) representations of the $\mathcal{N}=2$ algebra:

$$|B_0,\eta\rangle = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dP\ U^{\eta}(P,\omega)\ |P,\omega,NS;B,\eta\rangle\rangle\,. \tag{B.13}$$

In writing an integral over ω in (B.13), we are focusing on the case when the time direction is infinite in extent. To build non-BPS type II branes from these boundary states, GSO-invariance requires $U(P,\omega) = U^{+}(P,\omega) = -U^{-}(P,\omega)$ according to (B.10). We will therefore write

$$|\hat{D0}\rangle = |B_0,\eta=+1\rangle\rangle - |B_0,\eta=-1\rangle\rangle. \tag{B.14}$$

Next, we make the self-consistent assumption that the bosonic open-string spectrum should contain only states with Liouville momentum $P=-i$ corresponding to the identity state. Given (B.5) a solution to this requirement is [50, 51]

$$U(P,\omega) = \sqrt{2}\cdot e^{i\delta(P,\omega)}\cdot \frac{\sinh \pi P}{\cosh(\pi p/2 + i\pi\omega/2)}\,. \tag{B.15}$$

The phase $e^{i\delta(P,\omega)}$ is not actually determined by the modular hypothesis. As we will verify next, the D-brane with this wavefunction (B.15) indeed has only the identity representation in its bosonic open-string spectrum.

Open string spectrum. The vacuum annulus amplitude between the boundary state and itself,

$$\mathcal{A}(q) = \langle\hat{D0}|D(\tilde{q})|\hat{D0}\rangle\,, \tag{B.16}$$

determines the open string spectrum by channel duality. In this expresseion, $D(\tilde{q})$ is the closed-string propagator, on a tube of length $\tau = \ln\tilde{q}/2\pi i$. Given the expression (B.13), it takes the form

$$\mathcal{A}(q) = \int_{-\infty}^{\infty} dP\ d\omega\ \hat{U}(P,r)\ \hat{U}^{\dagger}(P,\omega) \sum_{\eta_1\eta_2} \eta_1\eta_2\ \chi^{NS}_{[P,\omega]}(\tilde{q},\eta_1\eta_2)\ \chi^{NS}_{gh}(q,\eta_1,\eta_2)\,. \tag{B.17}$$

Note that there are two sets of terms which are identical. This reflects the fact that unstable branes are $\sqrt{2}$ times heavier than BPS D-branes.

Using the modular transformation formulas above, and the wavefunction (B.15), in the open-string channel this reads

$$\mathcal{A}(q) = 2\left(\chi^{NS}_1(q,1)\ \chi^{NS}_{gh}(q,1) + \chi^{R}_1(q,1)\ \chi^{R}_{gh}(q,1)\right). \tag{B.18}$$

As usual for two-dimensional strings, the contribution from the $\mathcal{N}=2$ $bc\beta\gamma$ ghost system cancels all of the modular functions from each term, and we find

$$Z \equiv \int \frac{dt}{2t} \mathrm{Tr}\ q^{H_{\mathrm{open}}} = \int \frac{dt}{2t} \left[\left(q^{-1/2} - 2 + \cdots \right) - (1 + \cdots) \right] \tag{B.19}$$

with $q \equiv e^{-\pi t}$. Deducing the mass of the states propagating in the open string loop is subtle because the time direction participates in the $\mathcal{N}=2$ algebra, and the analytic continuation needs to be understood better. We can however understand the spectrum by looking at the large-t behaviour of the amplitude. The first term in brackets is the contribution of the NS sector. The two contributions indicated give rise to divergences and represent a tachyon Y, and a complex massless discrete state, respectively. This is similar to the case of the unstable D-particle in the bosonic $c=1$ string[6] (q.v. the lovely [3, appendix B]) and in the $\hat{c}=1$ type 0B string [7, 8].

The second term in brackets is the contribution of the R sector, which produces the fermions which were absent in the above examples. The term indicated represents a massless complex fermion ψ.

This is the spectrum of the gauged Marinari-Parisi model (2.2) on a circle. The appearance of non-quantized energies at intermediate stages can be remedied [51] by employing characters of the $\mathcal{N}=2$ algebra extended by the spectral flow generators.

For the instanton-anti-instanton pair, the only difference is that we use A-type Ishibashi states, which describe branes which are localized in the R-symmetry direction. The spectrum of the BPS D-instanton is obtained from this by an open-string GSO projection, and therefore has half as many fermion zeromodes, namely one.

Clearly we have merely begun to study the interesting zoology of D-branes in this system. Further development appears in [50, 51].

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